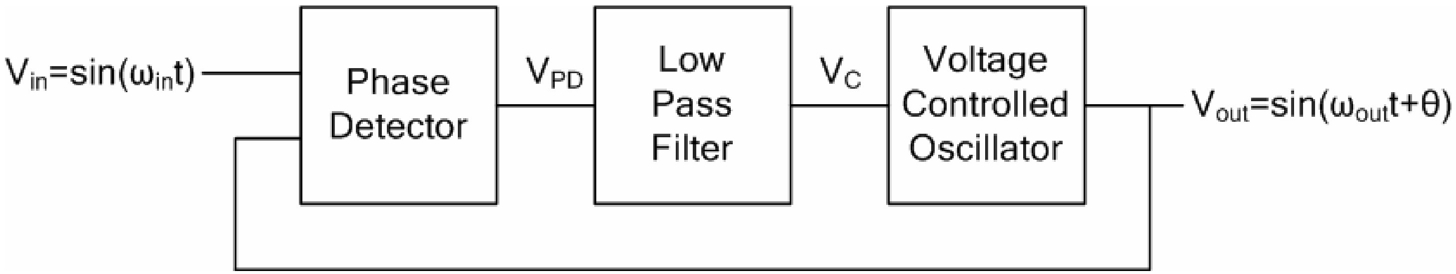
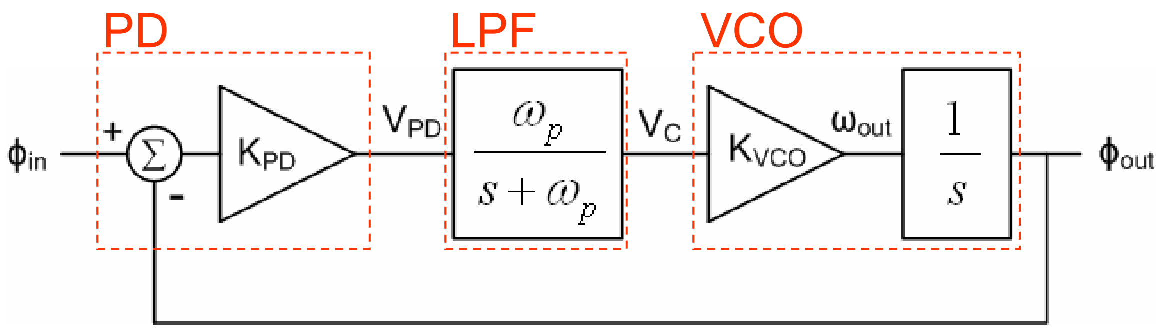
Lect. 23: PLL(2)

Linear PLL model:

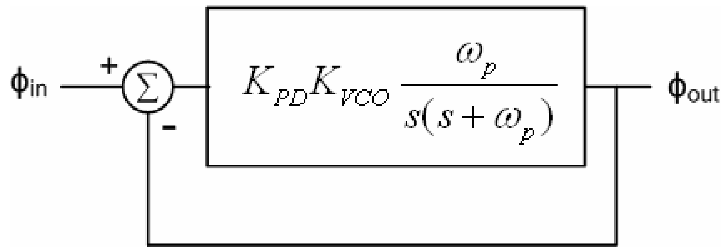
Block diagram



Linear model



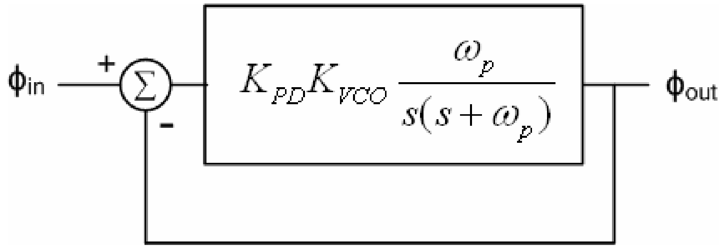
Simplified linear model



Lect. 23: PLL(2)

Open loop gain:

$$G(s) = K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}$$



Closed loop gain (transfer function): 2nd order low-pass filter

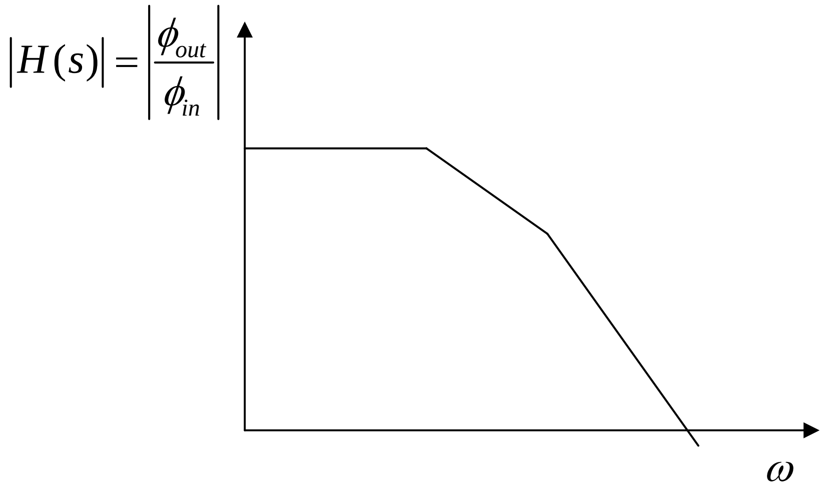
$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{G(s)}{1+G(s)} = \frac{K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}}{1+K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}}$$

$$= \frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$

Lect. 23: PLL(2)

Transfer function of PLL

$$H(s) = \frac{\phi_{out}}{\phi_{out}} = \frac{K_{PD} K_{VCO} \omega_p}{s^2 + \omega_p s + K_{PD} K_{VCO} \omega_p}$$

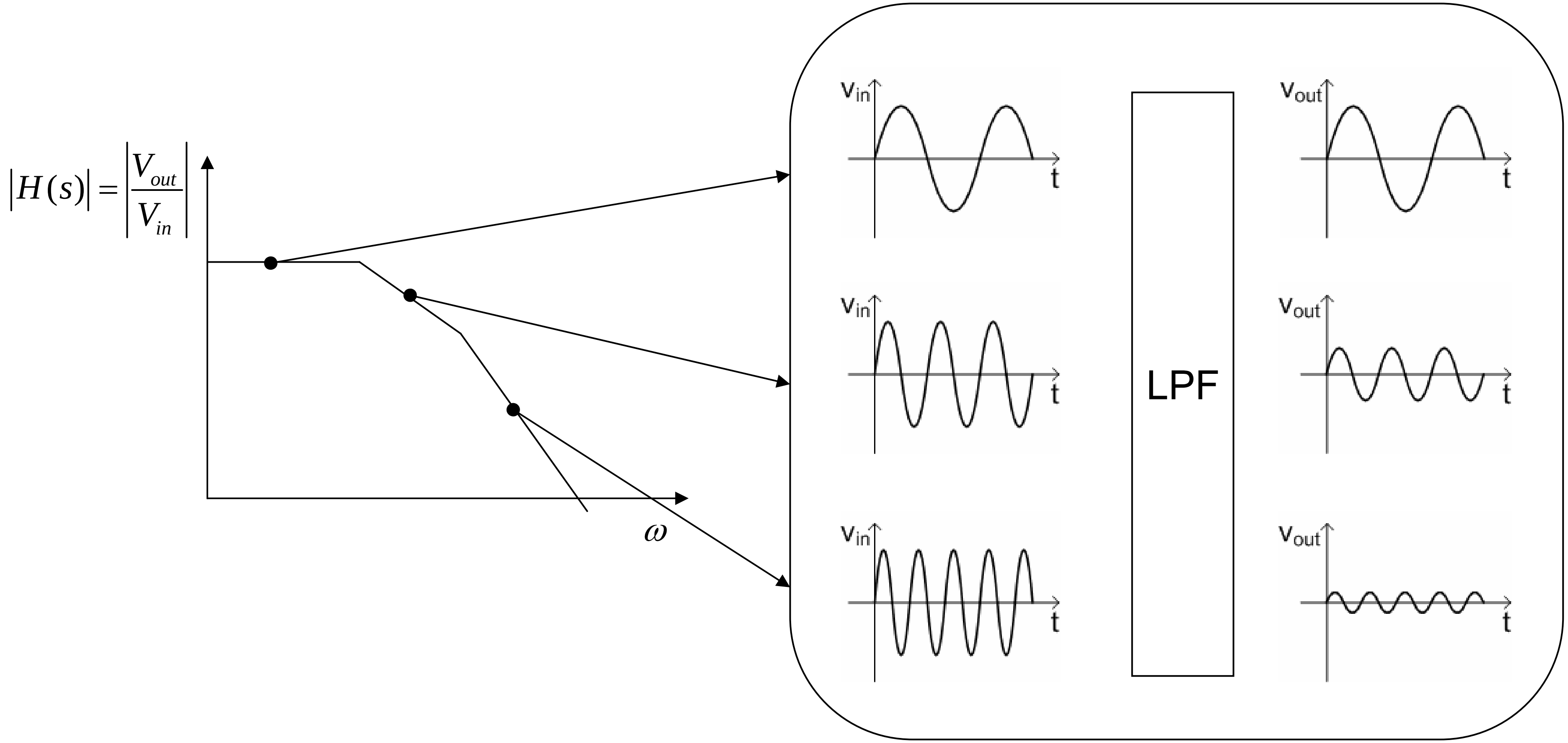


Note that input and output of PLL model are *'phase'*.

What does ω mean in x-axis?

Lect. 23: PLL(2)

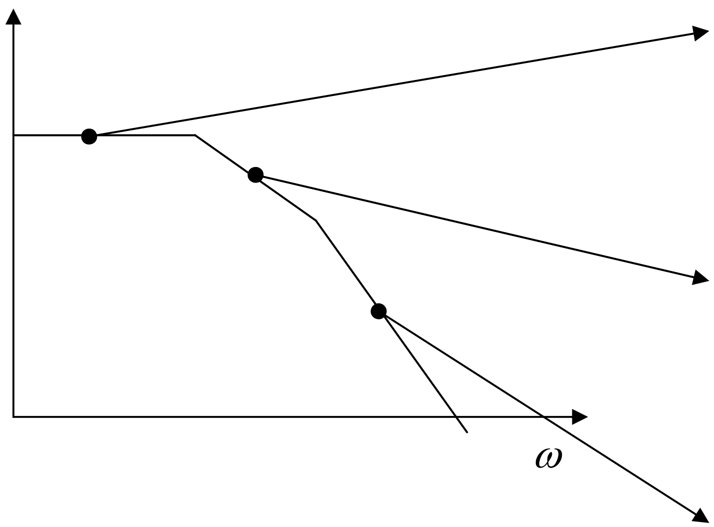
In LPF,

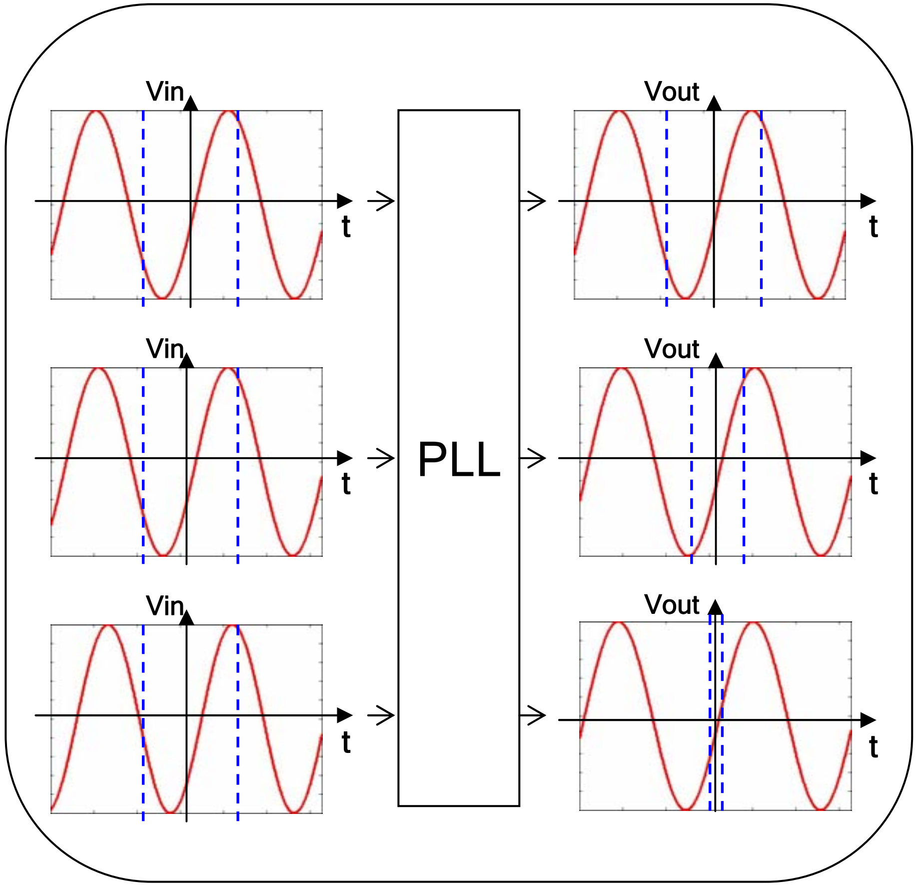


Lect. 23: PLL(2)

In PLL,

$$|H(s)| = \left|\frac{\phi_{out}}{\phi_{in}}\right|$$





Lect. 23: PLL(2)

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO}\omega_p}$$

In general 2nd order from,

$$H(s) = \frac{\omega^2{}_n}{s^2 + 2\zeta\omega_n s + \omega^2{}_n}$$

Natural frequency

$$\omega_n = \sqrt{\omega_P K_{PD} K_{VCO}}$$

Damping ratio

$$\zeta = \frac{1}{2}\sqrt{\frac{\omega_P}{K_{PD}K_{VCO}}}$$

Two poles

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta^2 - 1)\omega^2{}_n}$$

Lect. 23: PLL(2)

$$H(s)=\frac{\phi_{out}}{\phi_{in}}=\frac{\omega_{out}/s}{\omega_{in}/s}=\frac{\omega_{out}}{\omega_{in}}=\frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$

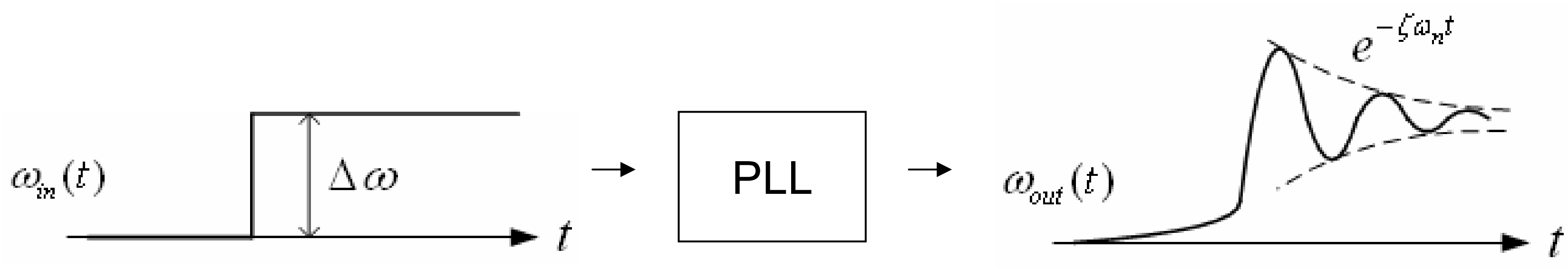
Step response

$$\omega_{in}(t)=\Delta\omega\cdot u(t)$$

$$\omega_{out}(t)=\left\{1-e^{-\zeta\omega_n t}\left[\cos(\omega_n\sqrt{1-\zeta^2}\cdot t)+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\cdot t)\right]\right\}\Delta\omega\cdot u(t)$$

$$=\left[1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\cdot t+\theta)\right]\Delta\omega\cdot u(t)$$

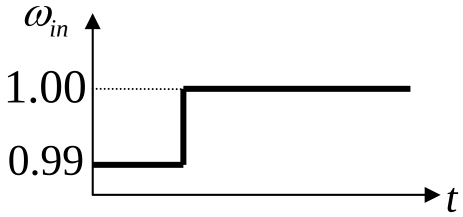
where $\theta=\sin^{-1}\sqrt{1-\zeta^2}$

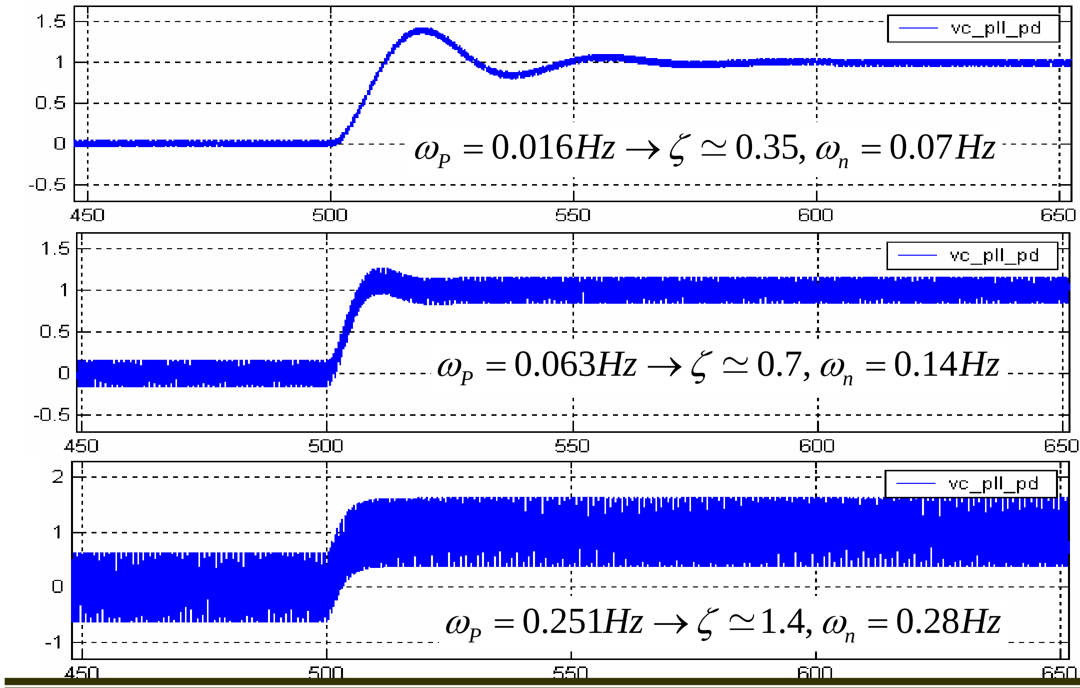


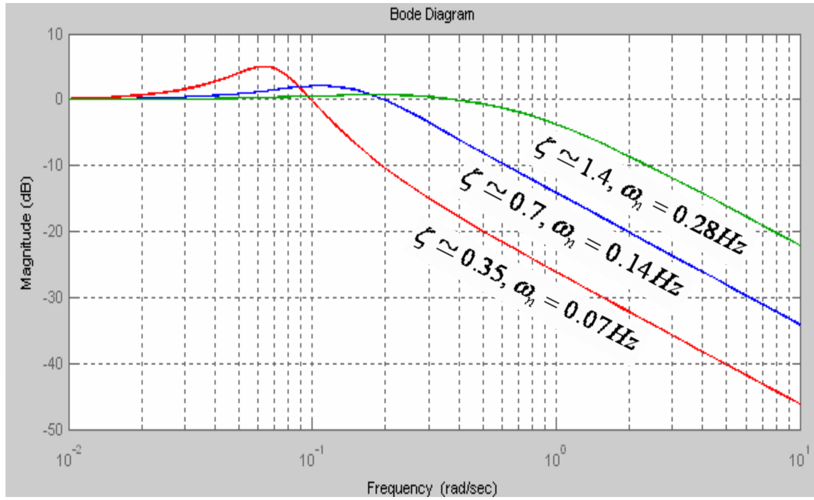
Lect. 23: PLL(2)

Step response simulation using CPPSIM

Simulation setup: $\omega_{in} = 1Hz,\ \Delta\omega_{in} = 0.01Hz,\ K_{PD} = 5V/rad,\ K_{VCO} = 2\pi \times 0.01 rad/s/V$







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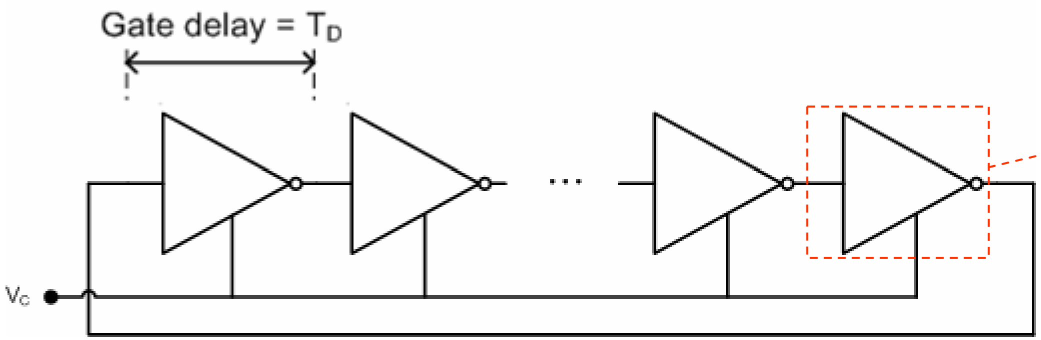
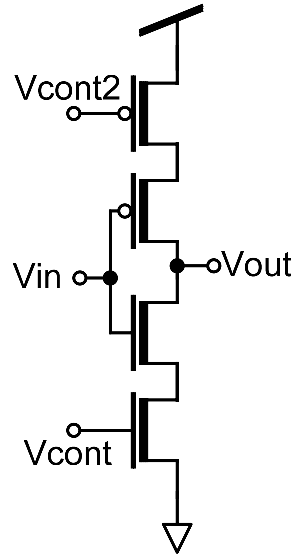
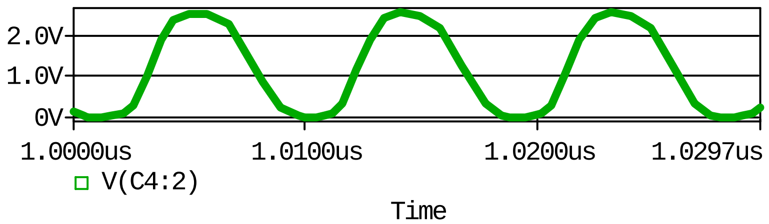
Building Blocks: VCO

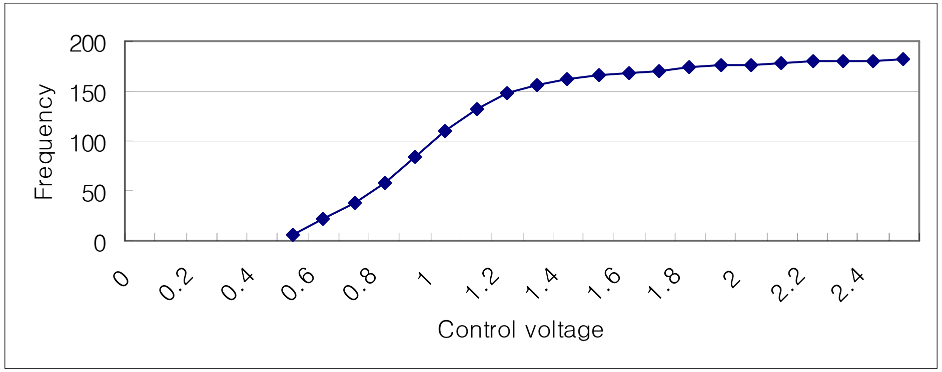
➔ Ring-oscillator (Odd-stage chain of inverters)





$$T = 2nT_D \qquad f = \frac{1}{T} = \frac{1}{2nT_D}$$





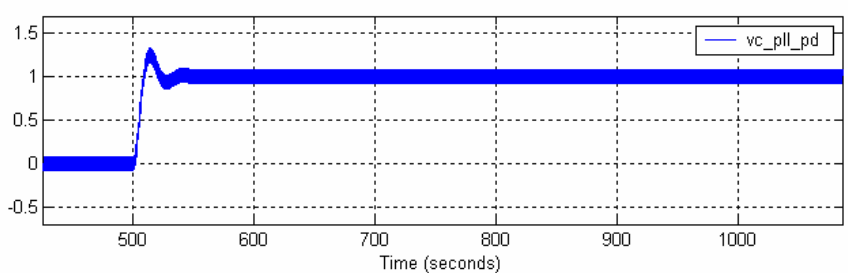
Lect. 23: PLL(2)

Problems of PLL using PD

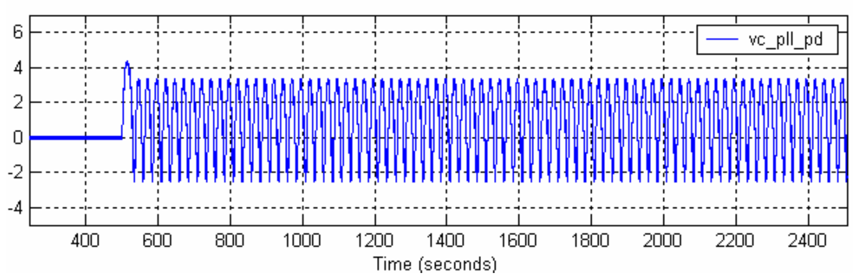
1. Narrow acquisition range → PLL acquisition range is roughly on the order of ω_P.

Simulation setup: $\omega_{in} = 1Hz,\ K_{PD} = 5V/rad,\ K_{VCO} = 2\pi \times 0.01 rad/s/V,\ and\ \omega_P = 0.032Hz$

$\Delta\omega_{in} = 0.01Hz$ (Locked)



$\Delta\omega_{in} = 0.05Hz$ (Lock failed)



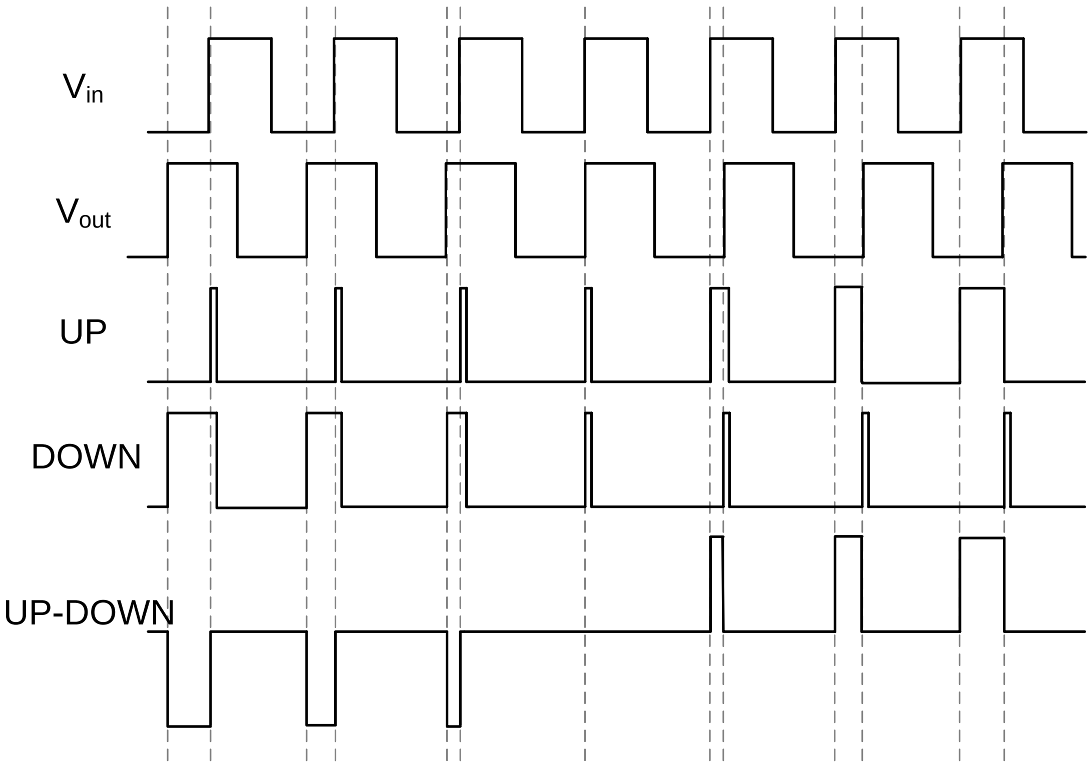
2. ζ and loop bandwidth (1st pole) can not be independently determined.

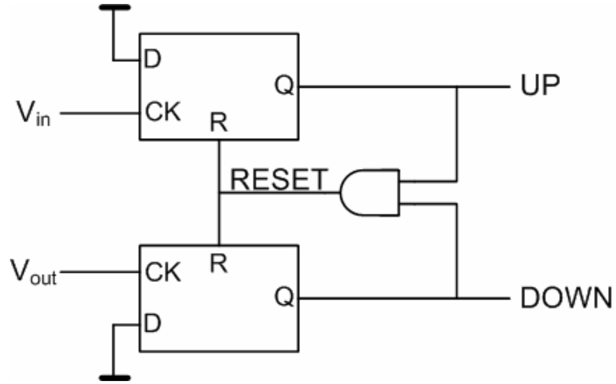
$$\omega_n = \sqrt{\omega_P K_{PD} K_{VCO}} \qquad \zeta = \frac{1}{2}\sqrt{\frac{\omega_P}{K_{PD} K_{VCO}}} \qquad s_1 = -\zeta\omega_n + \sqrt{(\zeta^2 - 1)\omega_n^2}$$

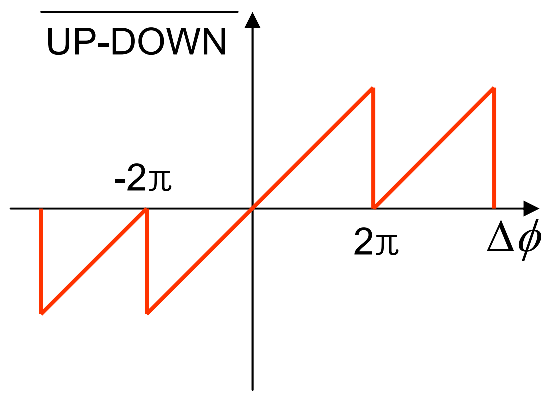
Lect. 23: PLL(2)

Solutions!

Phase and Frequency Detector (PFD)







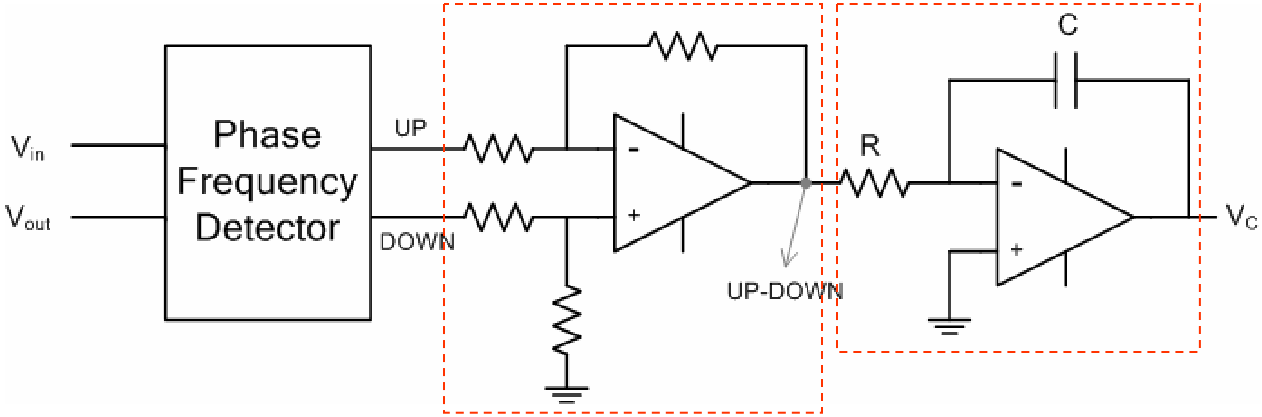
$\overline{\text{UP-DOWN}} \propto \Delta\phi$

Lect. 23: PLL(2)

Subtraction and integration

Voltage mode
using OP amplifiers

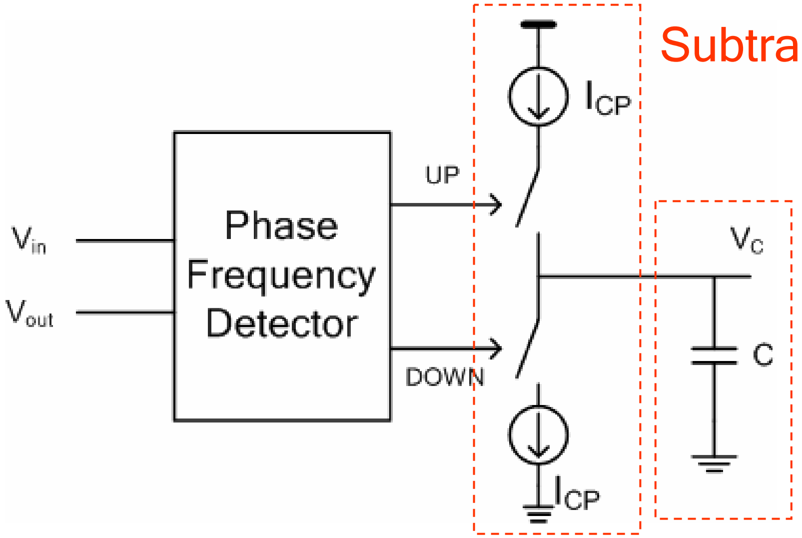
Subtraction Integration



Current mode
using charge pump

Subtraction

Integration



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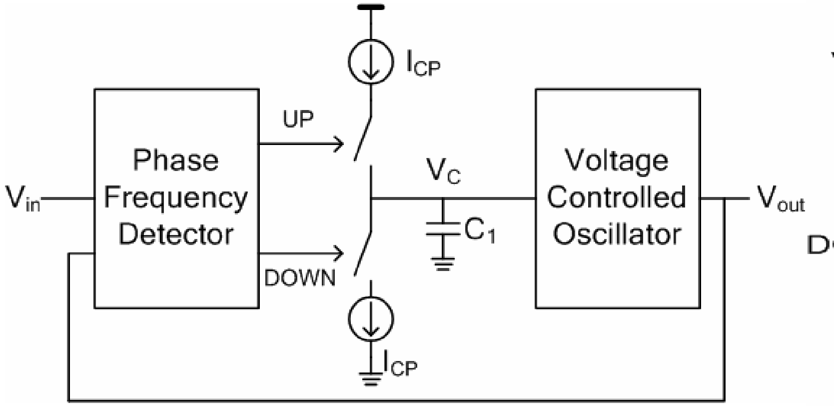
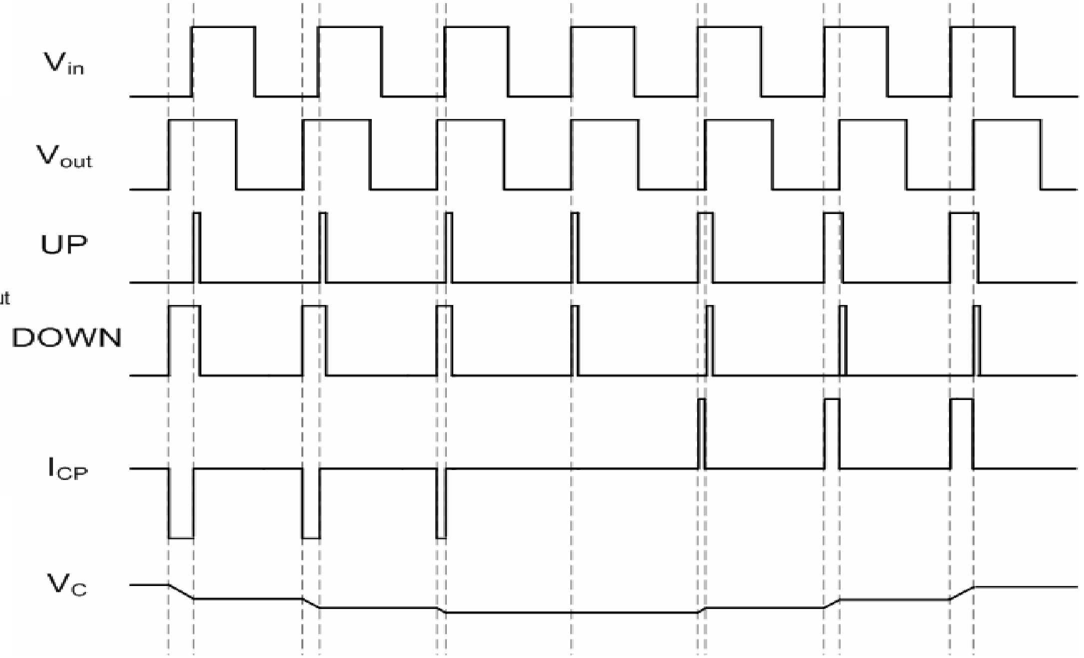
Lect. 23: PLL(2)

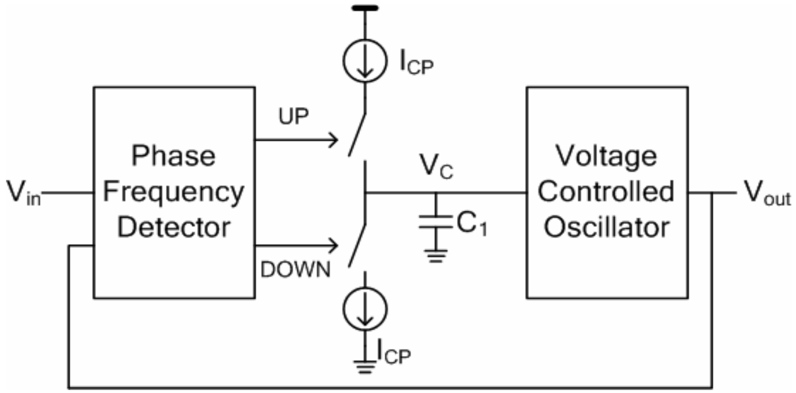
Charge Pump PLL

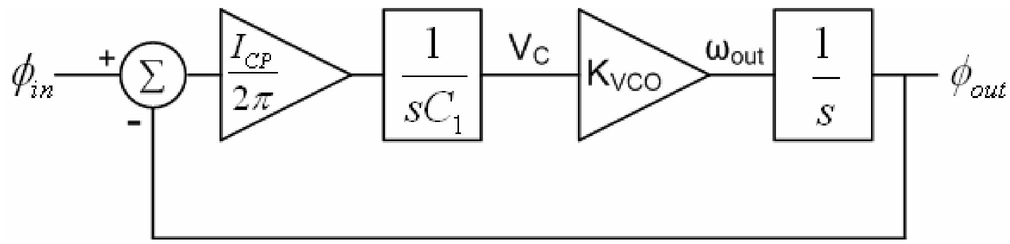




Lect. 23: PLL(2)

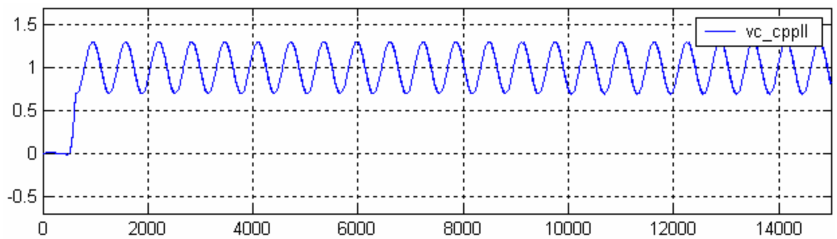
Linear model of CPPLL

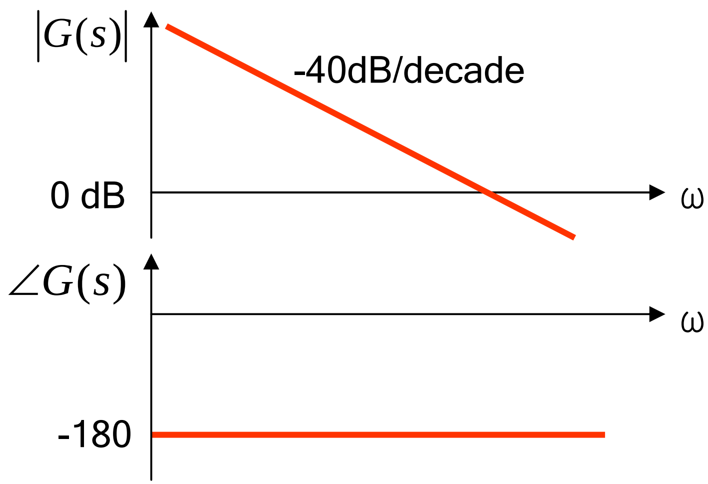




Open loop gain:

$$G(s) = \frac{1}{2\pi} I_{CP} K_{VCO} \frac{1}{s^2 C_1}$$

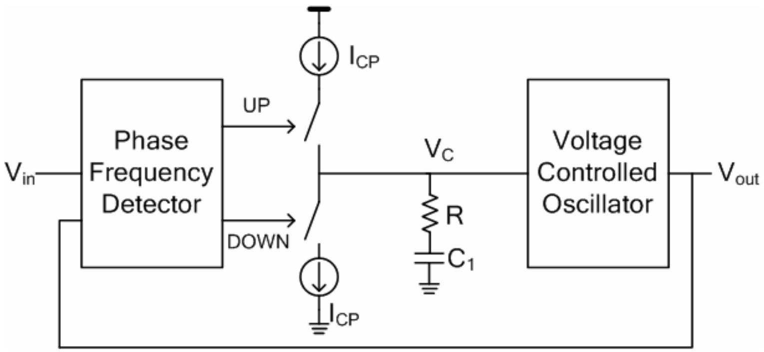


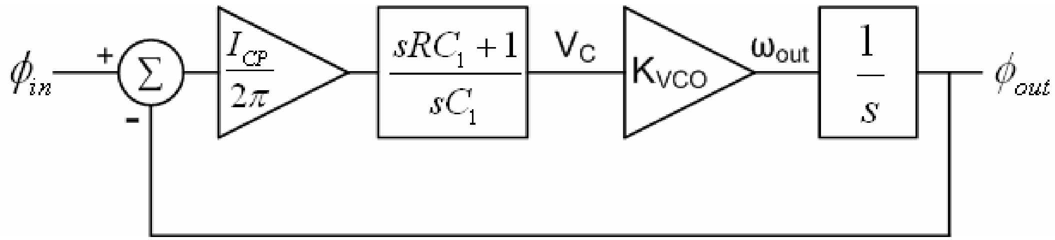


No phase margin → Unstable

Lect. 23: PLL(2)

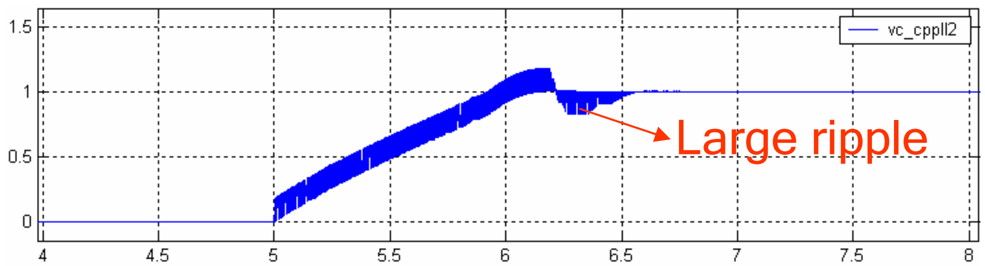
Charge Pump PLL

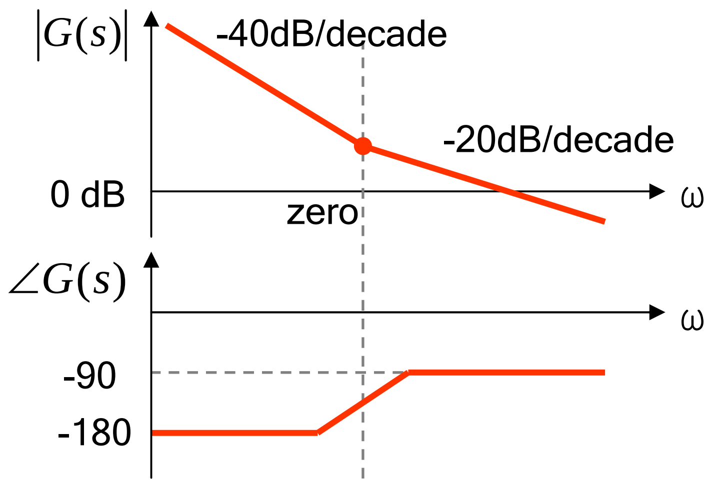




Open loop gain:

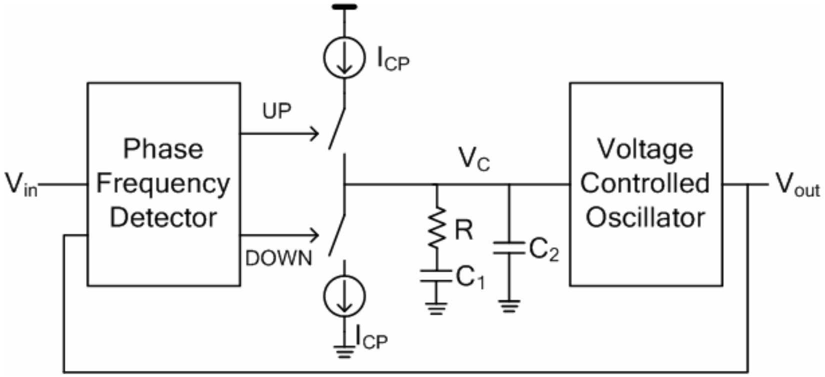
$$G(s) = \frac{1}{2\pi} I_{CP} K_{VCO} \frac{sRC_1 + 1}{s^2 C_1}$$

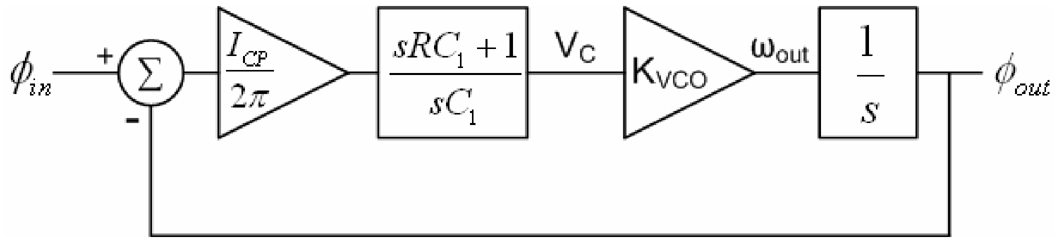




Lect. 23: PLL(2)

Charge Pump PLL



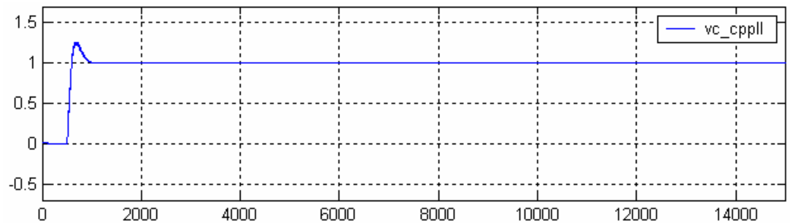


➔Ripple reduction with small C_2 ($\approx C_1/10$)

➔Simplification as 2nd-order system

Open loop gain:

$$G(s) = \frac{1}{2\pi} I_{CP} K_{VCO} \frac{sRC_1 + 1}{s^2 C_1}$$



Lect. 23: PLL(2)

Closed loop transfer function

$$H(s) = \frac{\frac{I_{CP}}{2\pi C_1} K_{VCO} (RC_1 s + 1)}{s^2 + \frac{I_{CP}}{2\pi} K_{VCO} Rs + \frac{I_{CP}}{2\pi C_1} K_{VCO}}$$

Natural frequency $\omega_n = \sqrt{\frac{I_{CP} K_{VCO}}{2\pi C_1}}$, K_{VCO} in rad/s/V and ω_n in rad/s

Damping ratio $\zeta = \frac{R}{2}\sqrt{I_{CP} C_1 K_{VCO}}$

➔ Independent change of natural frequency and damping ratio!

Lect. 23: PLL(2)

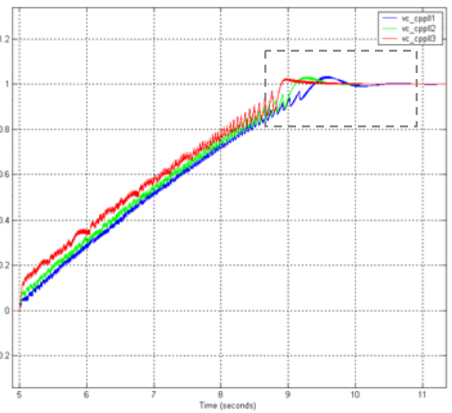
Transient simulation for various damping ratio and fixed ω_n

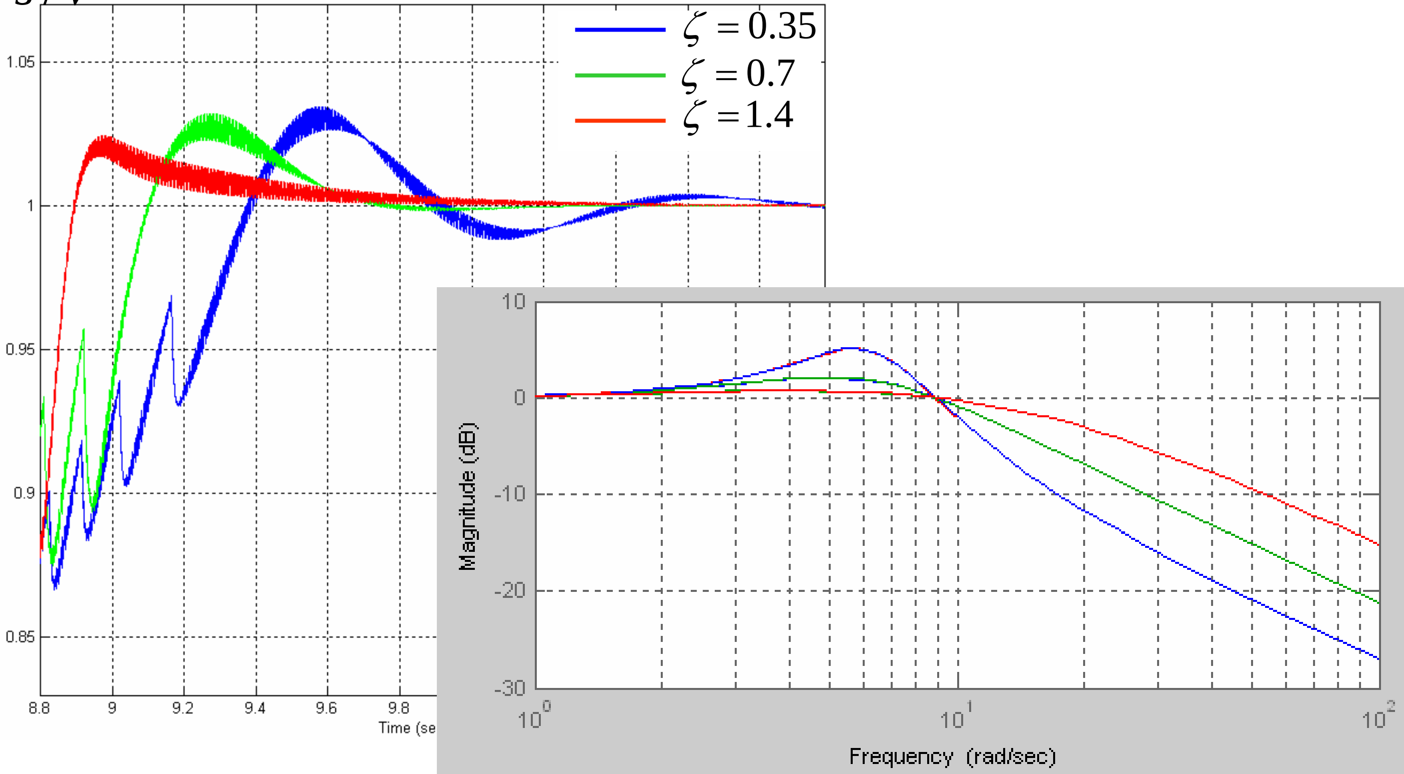
$$\omega_{in} = 200Hz$$

$$I_{CP} = 100\mu A$$

$$K_{VCO} = 2\pi \times 100 rad/s/V$$

$$\omega_n = 1Hz$$





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Lect. 23: PLL(2)

Transient simulation for various ω_n and fixed damping ratio

$$\omega_{in} = 200Hz$$

$$I_{CP} = 100\mu A$$

$$K_{VCO} = 2\pi \times 100 rad/s/V$$

$$\zeta = 0.7$$

➔ Optimization for desired performance!